Common features of vortex structure in long exponentially shaped Josephson junctions and Josephson junctions with inhomogeneities

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We study vortex structure in three different models of long Josephson junctions: exponentially shaped Josephson junction and Josephson junctions with resistor and shunt inhomogeneities in barrier layer. Numerical calculations of the possible magnetic flux distributions and corresponding bifurcation curves have done. For these three models the critical curves "critical current-magnetic field" are constructed. We develop an idea of the equivalence of exponentially shaped Josephson junction and rectangular junction with distributed inhomogeneity and demonstrate that at some parameters of shunt and resistor inhomogeneities at the ends of the junction the corresponding critical curves are very close to the exponentially shaped one. Pacs: 05.45.+b74.50.+r 74.40.+k.
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An exponentially shaped Josephson junction (EJJ) has been recently suggested as a tunable flux-flow oscillator operating at frequencies above 100 GHz[1]. The exponential variation of the junction width provides better impedance matching with an output load and allows one to avoid the chaotic regimes inherent to rectangular junctions[2].

In this paper we study the stability of the vortices in long Josephson junctions (JJ) and develop an idea[3] that vortex structure in exponentially shaped JJ and in JJ with inhomogeneity at the applicable end of the junction have common features. The use of inhomogeneities might have some technological advantages for construction of flux flow oscillator.

In order to investigate the stability of bound states we solve the static non-linear boundary value problem together with corresponding Sturm-Liouville problem[4],[5]. The minimal eigenvalue of the Sturm-Liouville problem allows to make a conclusion about the stability of the corresponding vortices in JJ[3 –9].

The corresponding boundary value problem for static dimensionless magnetic flux $\varphi(x)$ in case of in-line geometry has the form

$$-\varphi_{xx} + j_C(x)\sin\varphi = 0, \tag{1}$$

$$\varphi_x(0) = h_e - \varkappa_l L\gamma,\ \varphi_x(l) = h_e + \varkappa_r L\gamma, \tag{2}$$

Here L is the length of the junction, h_e — external magnetic field, γ — external current. The parameters $\varkappa_l$ and $\varkappa_r$ ($\varkappa_r + \varkappa_r = 1$) characterize the means of injection of the external current. The existence of the inhomogeneity leads to the local change of the Josephson current. In this paper the following approximation of the amplitude $j_C(x)$ is used

$$j_C(x) = \begin{cases} 1+\kappa, & x \in \Delta, \\ 1, & x \notin \Delta. \end{cases} \tag{3}$$

Here parameter κ describes the portion of Josephson current through the inhomogeneity. At $\kappa > 0$ we have shunt, at $\kappa \in [-1, 0)$ — microresistive type of the inhomogeneity. The value $\kappa = 0$ corresponds to the homogeneous junction.

We calculate the bifurcation curves "critical current-external magnetic field" for static fluxon states and construct the corresponding critical curves for the JJ. The results of the numerical experiments in the in-line geometry are compared for three models: exponentially shaped Josephson junction (EJJ) and rectangular Josephson junction with resistor (RJJ) and shunt (SJJ) inhomogeneities at the ends of junction. We choose the length of the junction $L = 7$, the width of inhomogeneity $\Delta = 0.7$, and parameters $\varkappa_l = 1$ and $\varkappa_r = 0$. Fig. 1 shows the distribution of the internal magnetic field $\varphi_x(x)$ along the junction for the fluxon Φ^1 at the value of the external magnetic field $h_e = 1.4$ and the value of the minimal eigenvalue of the Sturm-Liouville problem $\lambda_0 = 10^{-4}$, i.e., just before the destruction of the fluxon by the external current γ. We note a qualitative coincidence of the dependencies for these three models. The same qualitative coincidence we have observed for the distribution of the Josephson current. A quantitative difference takes place near the ends of junction, where the amplitude of Josephson current in RJJ and SJJ-models is changed. The found values of the critical currents $\gamma_{cr}(EJJ) \approx 0.126$, $\gamma_{cr}(RJJ) \approx 0.121$, $\gamma_{cr}(SJJ) \approx 0.134$ at $h_e = 1.4$ are very close to each other.

In Fig. 2 we compare the curves "critical current-magnetic field" for EJJ and SJJ models. As we can see they are qualitatively coincide. The similar behaviour demonstrates the Josephson junction with resistive inhomogeneity at the end of the junction (RJJ model)[9].

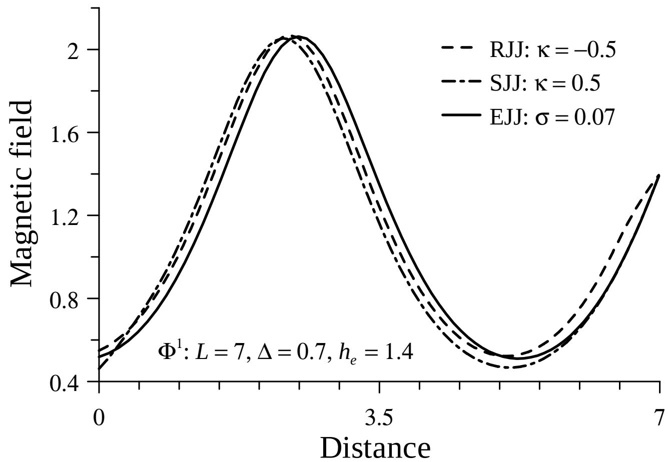

FIG. 1: Distribution of the internal magnetic field $\varphi_x(x)$ along the junction for the fluxon Φ^1 at $h_e = 1.4$ and $\lambda_0 = 10^{-4}$ for three models EJJ, RJJ and SJJ in the in-line geometry.

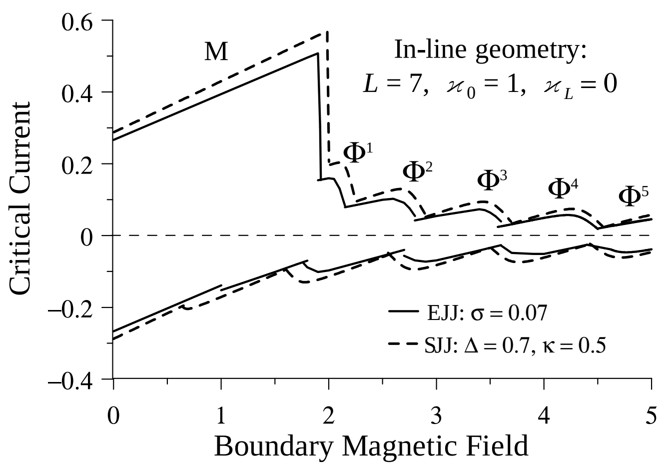

FIG. 2: Critical curves of the critical current versus magnetic field. The solid curve corresponds EJJ-model with $\sigma = 0.07$, the dashed curve corresponds to the SJJ-model with a rectangular shunted inhomogeneity ($\Delta = 0.7$, $\kappa = 0.5$

Both kind of inhomogeneities at the ends of the junction lead to the disappearance of the mixed fluxon-antifluxon states[5]. Possibly it explains the experimental results on the exponentially shaped Josephson junctions concerning the smaller linewidth of flux flow oscillator, increased output power and better impedance matching to a load. We consider that the use of the inhomogeneities might have a technological advantage for construction of the flux flow oscillator.

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